Debt sustainability arithmetic

This note presents a simple debt sustainability analysis to understand the dynamics at play between interest rates, GDP growth, primary deficits and debt sustainability.

Let $B_t$ be the level of nominal debt at time $t$, $Y_t$ be nominal GDP at time $t$, $D_t$ the primary deficit (excluding interest expense) at time $t$, and $r_t$ the net interest rate on government debt at time $t$.

As an accounting identity, the level of debt tomorrow is equal to the gross interest expense on the debt today plus any fiscal deficit:

$$B_{t+1} = (1 + r_t)B_t + D_{t+1}$$

Then dividing by $Y_{t+1}$ and rearranging gives us:

$$\frac{B_{t+1}}{Y_{t+1}} = (1 + r_t)\frac{B_t}{Y_t} \frac{Y_t}{Y_{t+1}} + \frac{D_{t+1}}{Y_{t+1}}$$

Now define $b_t = B_t/Y_t$ as the debt-to-GDP ratio at time $t$, and $g_t$ as the net growth rate of GDP at time $t$, and $d_t$ as the primary surplus (as a share of GDP) at time $t$. Noticing, for small values of $g$ and $r$, that $1 + \frac{r}{1+g} \approx 1 + r - g$ gives us the main equation for our analysis:

$$\Delta b_{t+1} = (r_t - g_t)b_t + d_{t+1}$$

It says that the change in the debt-to-GDP ratio is equal to the difference between the interest rate on debt and the growth rate of GDP times the current debt-to-GDP ratio plus the future deficit (% GDP).

Since our question is about sustainability, we care about what happens to debt-to-GDP in the long run. This is means we need to look at the steady state relationship:

$$\Delta b = (r - g)b + d$$

Note now that $\Delta b > 0$ means that debt-to-GDP grows forever, which is clearly not sustainable. So for debt-to-GDP to be sustainable, we need $\Delta b \leq 0$, i.e. not increasing in the long-run. So:

$$0 \geq \Delta b = (r - g)b + d$$
$$\implies -d \geq (r - g)b$$
$$\implies d \leq (g - r)b$$

This simple model tells us that for debt-to-GDP to be sustainable in the long run, the primary deficit (as a % of GDP) cannot be greater than the debt-to-GDP ratio times the
difference between the growth rate and the interest rate. If \( g > r \) – as is currently true in most of the developed world – then running a deficit in the long run is still consistent with sustainable debt, *as long as the deficit is not too large.*

Two features of this analysis are notable.

1. Should the fiscal authority, through a lack of fiscal discipline, let deficits increase above the sustainability threshold, the debt will become unsustainable *even though* \( r < g \).

2. The range of deficits consistent with debt sustainability is decreasing in \( r \). This means that a rise in interest rates – e.g. due to a rise in policy rates or an increase in the risk-premium associated with government bonds – may render a previously sustainable deficit now *unsustainable.*

This analysis makes clear that \( r < g \) is not a panacea to all debt sustainability concerns. It merely allows the government some leeway in engaging in a permanent but small deficit, whose size depends on \( r \). Certainly, a long-run primary deficit that violates this condition would still lead to an unsustainable debt burden on the economy and future generations.

We can use this model to calculate the maximum primary deficit (as a % of GDP) that the fiscal authority can run on average (\( d^* \)) without increasing the debt ratio. From the equation above, \( d^* = (g - r)b \). Table 1 shows \( d^* \) for the US, UK, Euro Area and Japan, setting the debt ratio \( b \) equal our post-pandemic (which we assume to be end-2021) gross debt level forecast.

For interest rates (\( r \)), we take the 2020-average of the bond yield with the closest tenor to the average maturity of outstanding debt for that country. It is 7 years for the US, 15 years for the UK, 7 years for the Euro Area and 10 years for Japan. For the Euro Area, we use the cross-sectional average of the AAA-rated government bond yields as provided by the ECB.

For GDP growth rates (\( g \)), we take the five-year ahead real GDP growth forecast from country-specific independent fiscal institutions where possible. For the US, our forecast is taken from the CBO. For the other countries, since independent fiscal institutions either do not exist or do not publish forecasts far enough into the future, we use forecasts from the IMF’s most recent WEO. We then add the 2% inflation target to calculate the nominal GDP growth rate for US, UK and Euro Area. For Japan, we set inflation at 1% – the bottom of the BoJ’s inflation target range – given the difficulty they have faced in raising inflation expectations thus far.

As Hall and Sargent (2011) clearly demonstrate, the precise \( r \) one should use is the average of the returns on each outstanding bond for the period in question. Since we are using the simple yield-to-maturity as a proxy for that more complicated object, and that the forward-looking estimates for nominal GDP and interest rates are subject to considerable uncertainty, these numbers should only be taken as ball-park figures.
Table 1. Maximum debt-sustainable primary deficits by country

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP growth (%)</th>
<th>Interest rate (%)</th>
<th>Debt ratio (% GDP)</th>
<th>Max deficit (% GDP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>3.6</td>
<td>0.75</td>
<td>137.2</td>
<td>3.9</td>
</tr>
<tr>
<td>UK</td>
<td>3.6</td>
<td>0.6</td>
<td>111.5</td>
<td>3.4</td>
</tr>
<tr>
<td>Euro Area</td>
<td>3.5</td>
<td>-0.6</td>
<td>107.6</td>
<td>4.4</td>
</tr>
<tr>
<td>Japan</td>
<td>2.0</td>
<td>0.0</td>
<td>264.0</td>
<td>5.2</td>
</tr>
</tbody>
</table>

Source: Haver, Country National Accounts, Central Bank Statistics

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